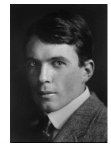
PHYSICS 534

ANSWERS

EXERCISE-20 **Suspended Objects**



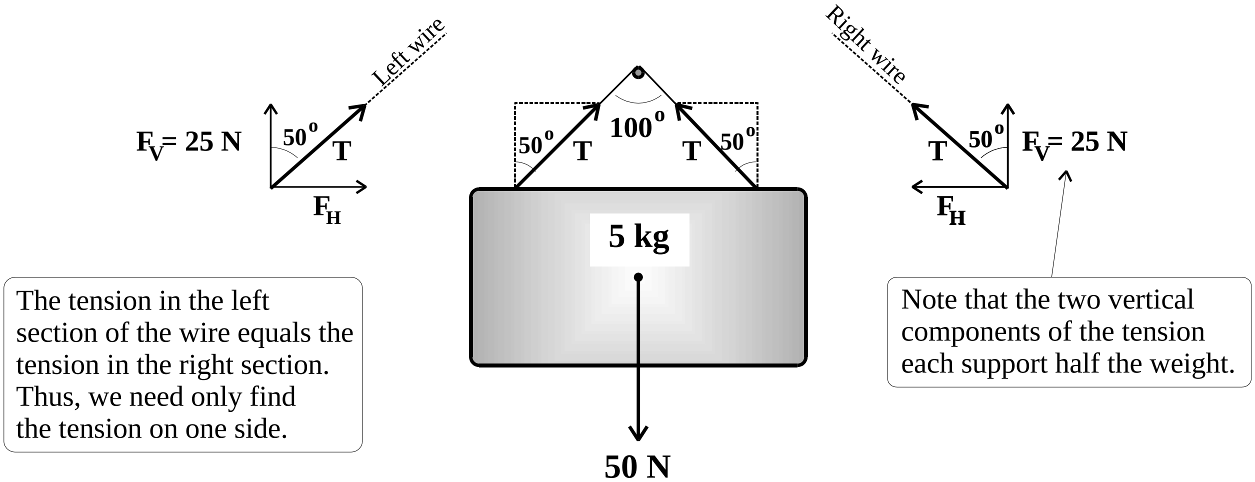
BRAGG

William Bragg junior received the Nobel prize for physics in 1915 for his X-ray analysis of crystal structure.

When objects hang motionless by way of a cord or wire they are in equilibrium and the force in the wire is called ***tension***.

When objects are suspended at an angle by a wire, the tension in the wire becomes greater as the angle increases.

Consider a picture (and frame) hanging on a wall. The wire holding the picture consists of two sections; a left wire and a right wire. Both sections are identical. The tension in each wire (or section) is the same. This is because the lengths of the two wires are equal.



To find the tension, we divide the wire into two parts, a left section and a right section. The tension on each section (wire) is the same because the lengths of the wires are equal. Thus, we need only find the tension on one side (wire).

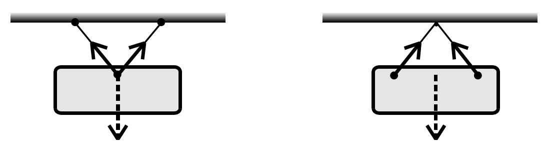
Also, note that each side consists of a right-triangle with the tension forming the hypotenuse. The angle in each triangle is ***half*** the angle at the "hook" ($100^o/2$ in the example above).

Since the system is at rest (F_R=0), we know that the total downward force equals the total upward force. But, the total downward force is the weight of the picture frame, 50 N. Thus, the total upward force must also be 50 N. This means the **vertical** components of the tension must each equal 25 N (50/2). Notice how the two (upward) vertical components each share the weight equally. Each right triangle has an angle of 50^o ($100^o/2$), an hypotenuse equal to the tension (T), and a vertical component (adjacent side) of 25 N (half the weight). Using trig, we can find the tension:

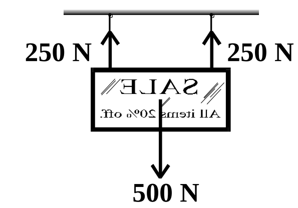
$$\text{Cos } 50^o = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{25\text{ N}}{\text{T}} \qquad \text{Thus:} \quad \text{T} = \frac{25\text{ N}}{\text{Cos } 50^o} = \frac{25\text{ N}}{0.642} = 38.9 = 39\text{ N}$$

The horizontal component forces (left and right) cancel each other (in accordance with the condition of equilibrium).

Finally, note that suspending an object by a wire using either two hooks or one hook results in the same tension as illustrated in the diagram on the right.



1. A 50 kg glass sign is suspended by two cords as shown in the diagram. Find the tension in each cord. [250 N]



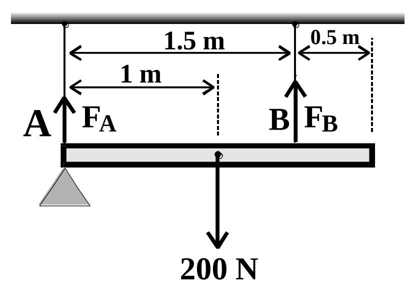
Since the system is in equilibrium, $\Sigma F_{UP} = \Sigma F_{DOWN}$

Thus, since $\Sigma F_{DOWN} = 500\ \text{N}$, $\therefore \Sigma F_{UP} = 500\ \text{N}$

Because each cord is erqually distant from the weight, each carries half the weight.

Answer: Each cord has a tension of 250 N

2. A 2 m long uniform beam having a mass of 20 kg is suspended by two cords (A and B) as shown in the diagram. Find the tension in each cord. [67 N] [133 N]



Take moments about end A.

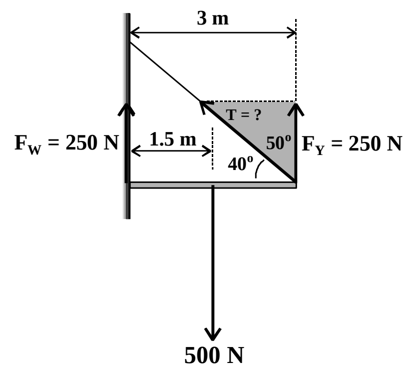
$\Sigma\ \text{cwm} = \Sigma\ \text{ccwm}$

$(200\ \text{N})(1\ \text{m}) = F_B(1.5\ \text{m})$

$$\therefore\ F_B = \frac{(200\ \text{N})(1\ \text{m})}{1.5\ \text{m}} = 133.3\ \text{N} = 133\ \text{N}$$

Thus, $F_A = 200\ \text{N} - 133\ \text{N} = 66.6\ \text{N} = 67\ \text{N}$

3. A 50 kg uniform plank is fixed to a wall and held horizontally by a wire as shown in the diagram. Find the tension in the wire. [389 N]



Since the system is in equilibrium, $\Sigma F_{UP} = \Sigma F_{DOWN}$

$\because\ \Sigma F_{DOWN} = 500\text{ N}, \quad \therefore \Sigma F_{UP} = 500\text{ N}$

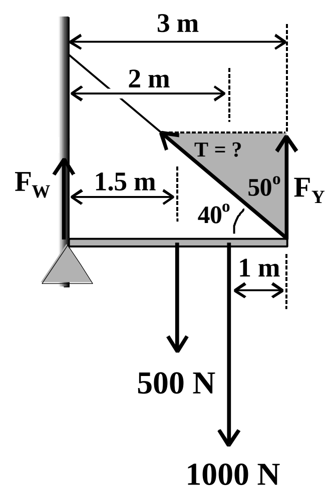
But $\Sigma F_{UP} = F_W + F_Y$ (wall force + vertical component of tension)

$\therefore\ F_W = F_Y = 250$ N (half the weight since each is equally distant from the weight)

Finally, with reference to the shaded triangle

$$\text{Cos } 50^\circ = \frac{250\text{ N}}{T} \quad \therefore\ T = \frac{250\text{ N}}{\text{Cos } 50^\circ} = 388.9\text{ N} = 389\text{ N}$$

4. A 50 kg uniform plank is fixed to a wall and held horizontally by a wire as shown in the diagram. A 1000 N weight is attached to the plank as indicated. Find the tension in the wire. [1427 N]



Find the vertical component of the tension (F_Y) by taking moments from the wall end.

$\Sigma \text{cwm} = \Sigma \text{ccwm}$

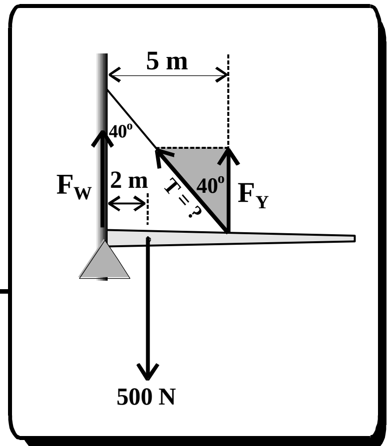
$(500\text{ N})(1.5\text{ m}) + (1000\text{ N})(2\text{ m}) = F_Y(3\text{ m})$

$$F_Y = \frac{(500\text{ N})(1.5\text{ m}) + (1000\text{ N})(2\text{ m})}{3\text{ m}} = 916.6\text{ N} = 917\text{ N}$$

Finally, with reference to the shaded triangle :

$$\text{Cos } 50^\circ = \frac{F_V}{T} = \frac{917\text{ N}}{T} \quad \therefore\ T = \frac{917\text{ N}}{\text{Cos } 50^\circ} = 1426.5\text{ N} = 2427\text{ N}$$

5. A tapered flag pole is suspended horizontally. As shown in the diagram, the thicker end is fixed to a wall and a wire is used to hold the pole horizontally. Knowing that the mass of the pole is 50 kg and that its center of gravity is 2 m from the thicker end, find the tension in the wire. [261 N]



Find the vertical component of the tension (F_Y) by taking moments from the wall end.

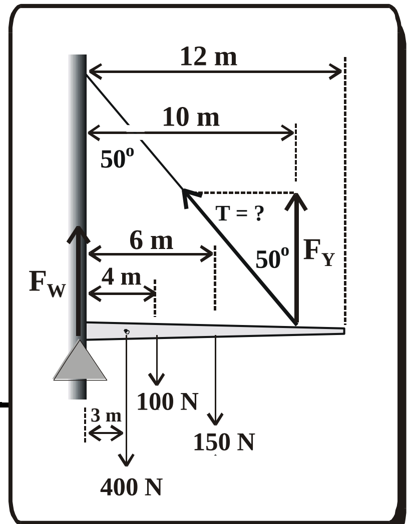
Σ cwm = Σ ccwm

$$(500\text{ N})(2\text{ m}) = F_Y(5\text{ m}) \quad \therefore\ F_Y = \frac{(500\text{ N})(2\text{ m})}{5\text{ m}} = 200\text{ N}$$

Finally, with reference to the shaded triangle:

$$\text{Cos } 40^o = \frac{F_Y}{T} = \frac{200\text{ N}}{T} \quad \therefore\ T = \frac{200\text{ N}}{\text{Cos } 40^o} = 261.1\text{ N} = 261\text{ N}$$

6. A 40 kg tapered stick is 12 m long and has its center of gravity 3 m from the thicker end. The thicker end is fixed to a wall and held horizontally by a wire. The wire is attached 2 m from the thinner end and makes an angle of 50° with the wall. Two weights of 100 N and 150 N are attached 4 m and 6 m respectively from the wall. Find the tension in the wire. [389 N]



Find the vertical component of the tension (F_Y) by taking moments from the wall end.

Σ cwm = Σ ccwm

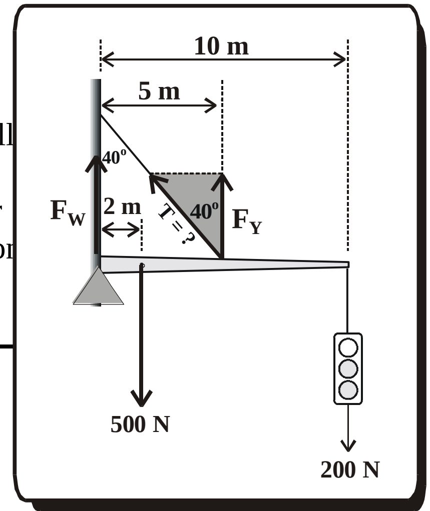
$$(400\text{ N})(3\text{ m}) + (100\text{ N})(4\text{ m}) + (150\text{ N})(6\text{ m}) = F_Y(10\text{ m})$$

$$F_Y = \frac{(400\text{ N})(3\text{ m}) + (100\text{ N})(4\text{ m}) + (150\text{ N})(6\text{ m})}{10\text{ m}} = 250\text{ N}$$

Finally, with reference to the shaded triangle:

$$\text{Cos } 50^o = \frac{F_Y}{T} = \frac{250\text{ N}}{T} \quad \therefore\ T = \frac{250\text{ N}}{\text{Cos } 50^o} = 388.9\text{ N} = 389\text{ N}$$

7. A tapered aluminum beam, 10 m long, is used to suspend a 20 kg traffic light above a street. The thicker end of the beam is attached to a hydro electric pole and is suspended horizontall by a wire attached in the middle of the beam (see diagram). Knowing that the mass of the beam is 50 kg and that its center of gravity is situated 2 m from the thicker end. Find the tensior in the wire. [783 N]



Find the vertical component of the tension (F_Y) by taking moments from the wall end.

Σ cwm = Σ ccwm

$$(500\text{ N})(2\text{ m}) + (200\text{ N})(10\text{ m}) = F_Y(5\text{ m})$$

$$F_Y = \frac{(500\text{ N})(2\text{ m}) + (200\text{ N})(10\text{ m})}{5\text{ m}} = 600\text{ N}$$

Finally, with reference to the shaded triangle:

$$\text{Cos } 40^o = \frac{F_Y}{T} = \frac{600\text{ N}}{T} \quad \therefore \; T = \frac{600\text{ N}}{\text{Cos } 40^o} = 783.2\text{ N} = 783\text{ N}$$

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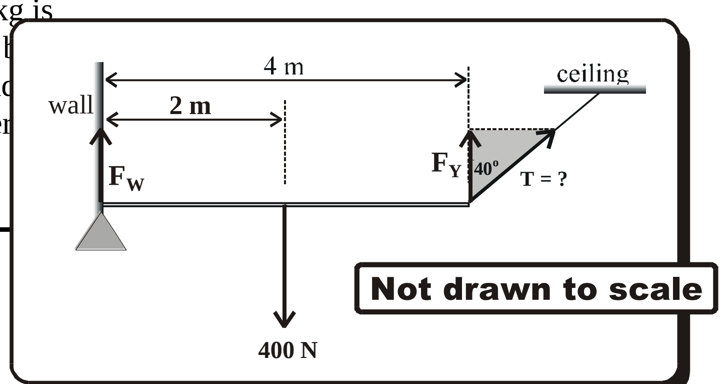
8. A uniform beam having a mass of 40 kg is suspended in a circus. One end of the b is attached to a wall while the other enc attached at an angle of 40° from the ve Find the tension in the wire. [261 N]



Find the vertical component of the tension (F_Y) by taking moments from the wall end.

Σ cwm = Σ ccwm

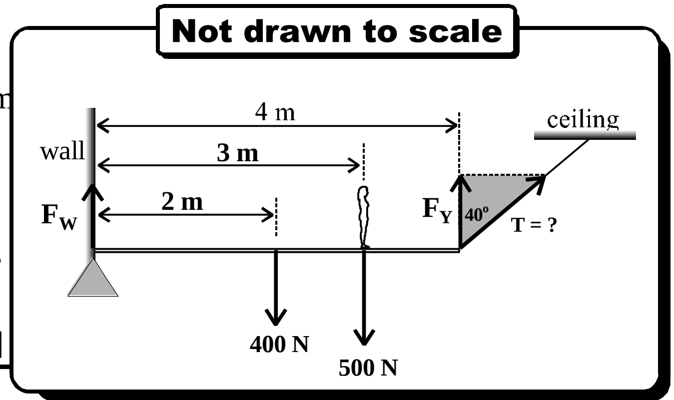
$$(400\text{ N})(2\text{ m}) = F_Y(4\text{ m})$$

$$F_Y = \frac{(400\text{ N})(2\text{ m})}{4\text{ m}} = 200\text{ N}$$

Finally, with reference to the shaded triangle:

$$\text{Cos } 40^o = \frac{F_Y}{T} = \frac{200\text{ N}}{T} \quad \therefore \; T = \frac{200\text{ N}}{\text{Cos } 40^o} = 261.1\text{ N} = 261\text{ N}$$

9. A uniform beam having a mass of 40 kg is suspended in a circus. One end of the beam is attached to a wall while the other end is attached to a ceiling by means of a wire making an angle of 40^o from the vertical. As indicated in the diagram below, a 50 kg acrobat stands 3 m from the wall.



Determine the tension in the wire. [751 N]

Find the vertical component of the tension (F_Y) by taking moments from the wall end.

Σ cwm = Σ ccwm

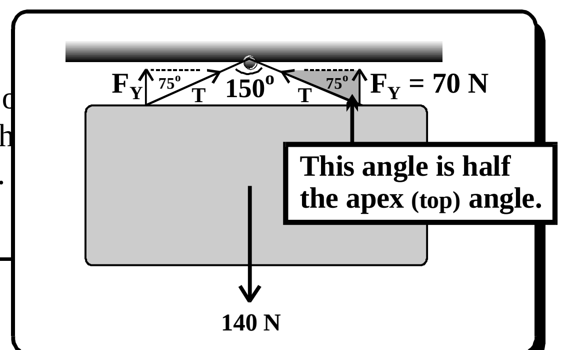
$(400\text{ N})(2\text{ m}) + (500\text{ N})(3\text{ m}) = F_Y(4\text{ m})$

$$F_Y = \frac{(400\text{ N})(2\text{ m}) + (500\text{ N})(3\text{ m})}{4\text{ m}} = 575\text{ N}$$

Finally, with reference to the shaded triangle:

$$\text{Cos } 40^o = \frac{F_Y}{T} = \frac{575\text{ N}}{T} \quad \therefore\ T = \frac{575\text{ N}}{\text{Cos } 40^o} = 750.6\text{ N} = 751\text{ N}$$

10. A 14 kg picture hangs on a wall by a wire passing over a hook as shown in the diagram on the right. Note that the angle between the two sections of the wire is 150^o. What is the tension in the wire? [270 N]

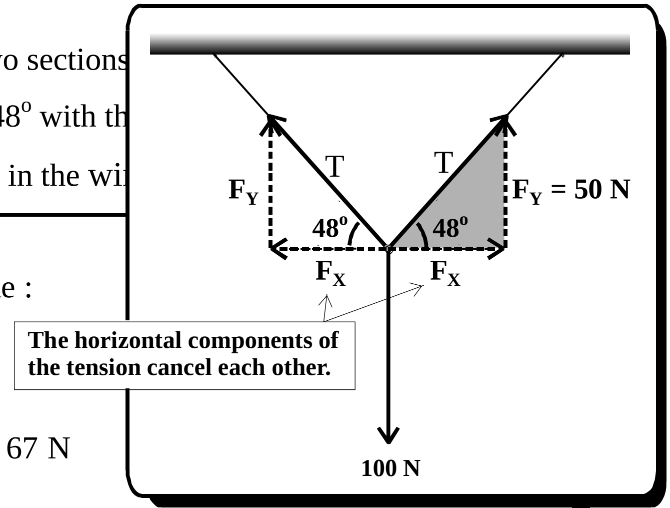


Note that the tension on either wire (left or right) is the same. Also, note that the vertical component of the tension (F_Y) on each side is half the weight.

Thus, with reference to the shaded triangle, we have:

$$\text{Cos } 75^o = \frac{F_Y}{T} = \frac{70\text{ N}}{T} \quad \therefore\ T = \frac{70\text{ N}}{\text{Cos } 75^o} = \frac{70\text{ N}}{0.2588} = 270.4\text{ N} = 270\text{ N}$$

11. A 10 kg mass is hung on a wire. The two sections the wire form symmetrical angles each 48° with th horizontal (see diagram). Find the tension in the wi



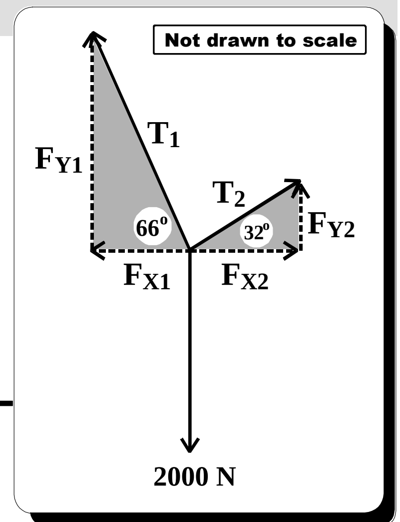
With reference to the shaded triangle :

$$\text{Sin } 48^o = \frac{F_Y}{T} = \frac{50 \text{ N}}{T}$$

$$\therefore \; T = \frac{50 \text{ N}}{\text{Sin } 48^o} = \frac{50 \text{ N}}{0.7431} = 67.2 \text{ N} = 67 \text{ N}$$

- **ENRICHMENT**

12. A 200 kg mass is attached to two wires as illustrated in the diagram below. Note that the angle each wire makes with the horizontal are 32° and 66° as shown. Find the tension, T_1 and T_2, of each wire. [1713 N] [822 N]



Since this system is in equilibrium, the up and down forces will cancel and the left and right forces will cancel. So we have:

1) $F_{X1} = F_{X2}$
2) $F_{Y1} + F_{Y2} = 2000N \rightarrow F_{Y2} = 2000N - F_{Y1}$

Using SOCAHTOA we have

$\tan 66 = \frac{F_{Y1}}{F_{X1}} \rightarrow F_{X1} = \frac{F_{Y1}}{\tan 66}$ and $\tan 32 = \frac{F_{Y2}}{F_{X2}} \rightarrow F_{X2} = \frac{F_{Y2}}{\tan 32}$

But from 1) $F_{X1} = F_{X2,}$ we have

$\frac{F_{Y1}}{\tan 66} = \frac{F_{Y2}}{\tan 32} \rightarrow \tan 32\,(F_{Y1}) = \tan 66\,(F_{Y2})$

From 2) $F_{Y2} = 2000N - F_{Y1}$, we have

$\tan 32\,(F_{Y1}) = \tan 66\,(2000 - F_{Y1})$
$0.625\, F_{Y1} = 4492.073 – 2.246\, F_{Y1}$
$2.871\, F_{Y1} = 4492.073$
$F_{Y1} = 1564.6$

$T_1 = \frac{1564.6}{\sin 66} = 1713$ N and $T_2 = \frac{(2000 - 1564.6)}{\sin 32} = 822$ N